Name: ____________________

Section: ____________________

Date: ____________________

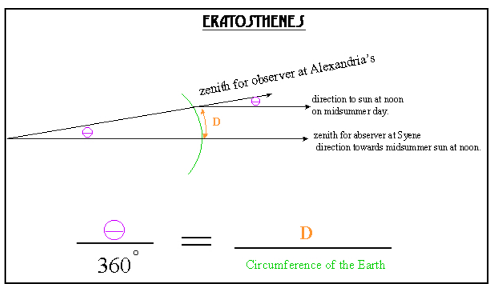
Prelab 4: Mass and Radius of the Earth

Measuring the Earth:

Since ancient times, those who have studied Astronomy have known the Earth is not flat. The first recorded measurement of the size of the Earth was made by a Greek astronomer named Eratosthenes in 240 BC. Eratosthenes measured the angle to the ground made by the sun at noon from two different cities in Egypt. Using geometry, he calculated the radius of the Earth to within kilometers of the presently accepted value.

Let's take a closer look at his methods: Eratosthenes knew that the Sun was very far away, so he could assume the sun's rays were approximately parallel. He also knew that on the summer solstice, the Sun is approximately directly overhead (read: "straight up"). From his location in Alexandria, he measured the shadows cast by the giant obelisk, allowing him to determine the angle the Sun made with the Earth at that location. He then repeated the experiment in Cyene. The final piece of information, the distance between Alexandria and Cyene, was determined with the help of a caravan of camels to be about (787 km). Using this data and some geometry, Eratosthenes was able to calculate the radius of the Earth.

How does the math work? Take a look at the diagram below:



As viewed from Earth, if the Sun goes all the way around the sky, its position changes by 360 degrees. Whatever fraction of angle (out of 360) the sun has moved from location to location must be equal to the fraction of the circumference of Earth we've traversed. Eratosthenes knew the distance between Alexandria and Cyene was 787 km and that the Sun's angle there was 90 degrees. He also measured the Sun's angle to be 82.8 degrees in Alexandria. Given all this, we can set up a ratio:

$$\frac{anglebetweencities}{360degrees} = \frac{distancebetweencities}{Circumference_{Earth}}$$

1. (3 points) According to his measurements, what did Eratosthenes measure the circumference of the Earth to be? Show your work.

2. (2 points) Given your answer above, what is the radius of the Earth?

3. (3 points) What are the actual circumference and radius of the Earth? How close was Eratosthenes?

Calculating the Mass of the Earth:

From Newton's laws describing the force of gravity, we know the acceleration of an object due to gravity is

$$Acceleration = \frac{(GravitationalConstant)\times(MassofPlanet)}{Radius^2_{planet}}$$
$$a_g = \frac{(G)\times(M_{planet})}{(R_{planet})^2}$$

The gravitational constant, G, is measured to have the value $6.67 \times 10^{-11} m^3 kg^{-1} sec^{-2}$. In cases where the acceleration is roughly constant, and some distance is traveled in a certain amount of time after starting from rest, the acceleration is given by:

$$Acceleration = \frac{2\times(DistanceTraveled)}{Time^2}$$
$$a = \frac{2\times(d)}{t^2}$$

We can use these two equations to obtain the mass of a planet in terms of the other quantities:

$$Mass_{planet} = \frac{2\times Radius^2\times(DistanceTraveled)}{Time^2\times GravitationalConstant}$$
$$M_{planet} = \frac{2\times R^2\times d}{t^2\times G}$$

This equation says that if we know the distance to the center of a planet, the distance an object on that planet falls in some time can be measured, and the mass of the planet can be obtained.

4. (2 points) Using the value of the gravitational constant G and the radius of the Earth you determined above, calculate the mass of the Earth in terms of the distance traveled and the time.

Picture Credits: `http://astrosun2.astro.cornell.edu/academics/courses/astro201/images/eratosthenes.gif`
Source material for fact-checking was found at `http://outreach.as.utexas.edu/marykay/assignments/eratos1.html`.

Name: ____________________

Section: ____________________

Date: ____________________

Lab 4: Mass and Radius of the Earth

From the prelab, we discovered:

$$Mass_{planet} = \frac{2 \times Radius^2 \times (DistanceTraveled)}{Time^2 \times GravitationalConstant}$$

$$\text{therefore, } M_{Earth} = \frac{1.23 \times 10^{24} \times (DistanceTraveled)}{t^2} \text{ kg.}$$

If we drop objects from some height, we can calculate how long it takes for the object to travel down and hit the ground, and plug it into the above equation; from this we can find the mass of the Earth.

1 Instructions

1. All height measurements should be in meters!
2. For the first set of drops, measure the height of your and your partner's waists. Partner 1 should hold the stopwatch and time how long it takes the ball to fall from waist-height to the ground. Partner 2 is responsible for dropping the ball from waist height. Fill in the table on the worksheet with the relevant information. Perform this measurement five times. Round the final answer of $\frac{Distance}{Time^2}$ to two significant figures.
3. Partner 1 becomes Partner 2 and vice-versa. Repeat the five measurements in step one.
4. For the second set of drops, measure your height and the height of your partner. If you and your partner are approximately the same height, have one partner perform the set of drops from shoulder height, and the other partner perform the set of drops from the top of his/her head. If there's a discrepancy between your height and your partner's, you may both drop the ball from the top of your heads.
5. Perform the Partner1-Partner 2 drops again, recording the appropriate information in the table on the worksheet. Take five measurements each. You should now have a total of twenty measurements recorded. Again, make sure you round the final answer of $\frac{Distance}{Time^2}$ to two significant figures.
6. Now you want to make a histogram of the data you just recorded. The chart should strongly resemble a bar graph. Label the x-axis of the plot in increments of one (this is the number of times (frequency) a specific data point occurs, see step 7). Your data should span a range of approximately 2-9, but if your data is slightly different, write the appropriate range along the x-axis.
7. Go through your data. For each measurement of $\frac{Distance}{Time^2}$ you measured, round to the nearest whole number, and place an x on the plot above the corresponding value. Each time you see that value again, stack another x above the previous one. The number of x's above each value should correspond to the number of times you measured that value. Your data will probably vaguely resemble a bell curve.
8. Complete the rest of the worksheet, following the instructions and calculations there.

Name: ______________________

Section: ______________________

Date: ______________________

Lab 4 Worksheet: Mass and Radius of the Earth

1. (5 points) Data Table

	Name	Distance	Time	$Time^2$	$\frac{Distance}{Time^2}$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

2. (5 points) Draw your histogram here (remember to label your axes!):

3. (2 points) Are your values spread out very thinly? Are they clumped together around different areas? Are they clumped around a single value? In detail, explain why you think your data looks the way it does.

4. (2 points) Use your histogram to pick the value for $\frac{Distance}{Time^2}$ which best represents your data. Write your answer below, and explain why you picked it.

5. (2 points) Using your histogram, estimate what you think your error in $\frac{Distance}{Time^2}$ might be. In other words, estimate on average how far your points are away from the value you chose as the value for $\frac{Distance}{Time^2}$.

6. (2 points) Plug your value for $\frac{Distance}{Time^2}$ into the equation for the mass of the Earth. Also plug your estimated error into this equation to find the error in your measurement of the mass of the Earth. Present your answer here in the form $(MassoftheEarth) = Value \pm Error$.

7. (2 points) Astronomers have calculated the mass of the Earth to be 5.97×10^{24} kg. Does your answer lie near this value? Is this value within your estimated error? Why or why not?