Name: _____

Section: _____

Date: _____

PHYS 133 Math Review - Winter 2015

Instructors:
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Your lab TA is here to help! Feel free to ask me questions and contact me anytime via email if you have questions or concerns about the class or your work. We understand that this can be a challenging course, and are happy to do our best to assist you.

1 Concepts

This course requires the use of several concepts from high school algebra that you may not have seen or used in many years. This worksheet has been designed to help remind you how to round, use scientific notation, and write answers in significant figures, as well as to cover math concepts such as exponent rules, conversion factors, and basic trigonometry [1].

1.1 Rounding

In general, rounding rules are fairly straightforward:

1. For numbers 0-4: round down.

2. For numbers 5-9, round up.

You need to know the *place* to which you are rounding. Take for example the number

$$123, 456.789 \, 0$$

The number 1 is in the *hundred-thousands* place.
The number 2 is in the *ten-thousands* place.
The number 3 is in the *thousands* place.
The number 4 is in the *hundreds* place.
The number 5 is in the *tens* place.
The number 6 is in the *ones* place.
The number 7 is in the *tenths* place.
The number 8 is in the *hundredths* place.
The number 9 is in the *thousandths* place.
The number 0 is in the *ten-thousandths* place.

If we want to round our number to the tenths place, we look at the number in the hundredths place, an eight. The rounding rules above state that we round eights up, and so the 7 in the tenths place becomes an 8. Our number is now 123, 456.8.

If we want to round our number to the thousands place, we check the number in the hundreds place, here a 4. According to the rounding rules, 4 gets rounded down, so we don't change the number 3 in the thousands place. We truncate the rest of the number at the thousands place, and fill in zeros before the decimal so that the thousands place retains its value. Our number becomes 123, 000.000 0.

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[1]Note the "Exponent Rules": and "Scientific Notation" portions of this worksheet are closely adapted from Appendix C of Bennett, Jeffrey O., Megan Donahue, Nicholas Schneider, and Mark Voit. The Essential Cosmic Perspective. 4th ed. San Francisco: Pearson/Addison Wesley, 2008. Print.

1.2 Exponent Rules

Exponents represent the number of times you multiply a number or quantity by itself. Multiplied quantities with the same base have added exponents; divided quantities have subtracted exponents:

$$2^2 \times 2^3 = 2^5$$
$$(1+x)^{12}/(1+x)^9 = (1+x)^3$$

Exponents with different bases have to be evaluated before multiplication, division, addition, or subtraction operations (recall your PEMDAS rules from elementary and middle school!).

If you want to raise an exponentiated number to another exponent, multiply the two exponents together:

$$(10^4)^5 = 10^{20}$$

Any base to the 0th power returns 1. Any base raised to the 1st power returns the base:

$$x^0 = 1$$
$$x^1 = x$$

Powers of ten are one particular application of exponents that generally can be divided into two categories:

1. **Positive** exponents represent the number of zeros added to the *right* of the original number.

2. **Negative** exponents represent the number of zeros added to the *left* of the original number.

In general, when we aren't dealing with powers of 10, negative exponents represent reciprocals, that is, one over the given base:

$$x^{-5} = \frac{1}{x^5}$$

Finally, Roots are just fractional powers, so we can write a square root as follows:

$$2^{\frac{1}{2}} = \sqrt{2}$$

The multiplication-addition and division-subtraction rules for fractional powers are the same:

$$2^{\frac{1}{2}} \times 2^{\frac{1}{4}} = 2^{\frac{3}{4}}$$
$$3^3 \div 3^{\frac{-3}{2}} = 3^{\frac{9}{2}}$$

1.3 Prefixes

Prefixes are a "short-hand," a way of writing powers of ten (and therefore scientific notation) in words. They are placed at the front of a unit and describe how many powers of ten are attached to the end of the listed number.

The following prefixes will be used regularly in lab this semester; you will want to memorize these and become comfortable with using them:

- G (giga) is one billion, that is 10^9

- M (mega) is one million, that is 10^6

- k (kilo) is one thousand, that is 10^3

- c (centi) is one hundredth, that is 10^{-2}

- m (milli) is one thousandth, that is 10^{-3}

- μ (micro) is one millionth, that is 10^{-6}

- n (nano) is one billionth, that is 10^{-9}

Example 1: Write 2.7×10^6 parsecs using a prefix.

Answer 1: 2.7 Mpc

Example 2: Write 4.8 centimeters in scientific notation.

Answer 2: 4.8×10^{-2}

1.4 Scientific Notation

Scientific notation is an easy way to write out long numbers. It proves particularly useful with exponents, simplifies calculations, and makes your work easier to read. In this class, use of scientific notation is a must. To convert a number into scientific notation, first move the decimal point until it's placed to the right of the first non-zero digit. The number of decimal places moved will be the power of ten you use. Moving the decimal point to the left corresponds to a positive power of ten; similarly, moving the decimal point to the right corresponds to a negative power of ten:

$$1945 = 1.945 \times 10^3$$
$$0.001945 = 1.945 \times 10^{-3}$$

Converting a number out of scientific notation uses the opposite convention: moving the decimal point to the left corresponds to a negative power of ten; similarly, moving the decimal point to the right corresponds to a positive power of ten. If needed, use zeros as place markers:

$$2.945 \times 10^{-3} = 0.002945$$
$$7.014 \times 10^2 = 701.4$$

Operations in scientific notation are similar to those with normal notation. To add and subtract, you usually need to switch out of scientific notation (or use the appropriate function on a calculator); however, if the numbers you are adding or subtracting have the same power of ten attached to them, you can factor the power of ten out (see example below). To multiply and divide, separate the powers of ten from the rest of the number and compute the two parts separately:

$$(1.2 \times 10^2) + (1.4 \times 10^2) = 1.6 \times 10^2$$
$$(3 \times 10^2) \times (4 \times 10^4) = 12 \times 10^6$$
$$(8 \times 10^{-2})/(2 \times 10^5) = 4 \times 10^{-7}$$

1.5 Significant Figures

Significant figures are important when performing calculations, as they reflect the precision of the measurement taken or data reported. The following list, borrowed from the University of South Carolina at Aiken's General Chemistry class[2], sums up the basic rules needed for working with significant figures in a lab setting:

1. All non-zero numbers are always significant.

2. All zeros between non-zero numbers are always significant.

3. All zeros which are simultaneously to the right of the decimal point and at the end of the number are always significant.

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[2] "Significant Figures - Rules." Significant Figures - Rules. University of South Carolina Aiken, n.d. Web. 23 Aug. 2014. `<http://www.usca.edu/chemistry/genchem/sigfig.htm>`,`"SignificantFigures-MathematicalOperations.` `"SignificantFigures-MathematicalOperations.UniversityofSouthCarolinaAiken,n.d.Web.23Aug.2014.<http://www.usca.` `edu/chemistry/genchem/sigfig2.htm>`.

4. All zeros which are to the left of a written decimal point and are in a number greater than or equal to ten are always significant.

5. If you can write the number in scientific notation and get rid of the zeros, the zeros are NOT significant.

6. When adding or subtracting numbers, count the number of decimal places to determine the number of significant figures. The answer cannot contain more places after the decimal point than the smallest number of decimal places in the numbers being added or subtracted.

7. When multiplying or dividing numbers, count the number of significant figures. The answer cannot contain more significant figures than the number being multiplied or divided with the least number of significant figures.

Here are some examples:

$$0.0369 \text{ has three significant figures}$$
$$107.3 \text{ has four significant figures}$$
$$27.0 \text{ has three significant figures}$$
$$0.003070 \text{ has four significant figures}$$
$$35.7 + 0.09 + 10.1 + 1100 = 1145.89, \text{ which we round to } 1100 \text{ to have two significant figures.}$$

1.6   Conversion Factors

Conversion factors are a way to switch between units. They allow you to keep track of all your steps (and powers of ten) in one line!
Example: How many kilometers are in a centimeter? Even if this conversion initially looks intimidating, we can break it down into pieces that are more easily understandable. Let's first set up our conversion factor, placing the units we are trying to get to in the numerator, and those we are trying to remove in the denominator:

$$(1 \text{ kilometer}) = \frac{?meters}{?kilometers} \times \frac{?centimeters}{?meters} = ? \text{ centimeters}$$

Now, look at the prefixes attached to the unit "meters:" at the beginning, we have "kilo," which refers to $1000 = 10^3$, and at the end we have "centi," which means $.01 = 10^{-2}$. Let's plug those in to the expression we have above:

$$(1 \text{ kilometer}) = \frac{10^3 meters}{1 kilometer} \times \frac{10^2 centimeters}{1 meter} = 10^5 \text{ centimeters}$$

As shown, once you have the prefixes plugged in, your conversion factor is complete, and all you have to worry about is multiplying powers of ten. Note that all the units, except the one you're trying to get to, cancel out! Just add the exponents and you're done.

Let's try another example: how many seconds are in a year? See if you can follow each conversion:

$$(1 \text{ year}) \times \frac{365.25 days}{1 year} \times \frac{24 hours}{1 day} \times \frac{60 minutes}{1 hour} \times \frac{60 seconds}{1 minute} = 31,557,600.00 \text{ seconds} = 3.15576 \times 10^7 \text{ seconds}$$

One final example: how many parsecs (pc) are in a lightyear (ly)? How many lightyears are in an AU? The first thing to realize is that the units AU, lightyears, and parsecs are all units of distance. The next step is to convert these foreign-looking units into something we recognize (it's ok if you have to look up some of these numbers):

$$1 AU = 1.496 \times 10^{11} \text{ meters}$$
$$1 parsec = 3.086 \times 10^{16} \text{ meters}$$

Now that we have these conversions in front of us, let's set up the conversion factors:

$$1ly = \frac{2.998 \times 10^8 meters}{1 second} \times \frac{3.15576 \times 10^7 seconds}{year} \times (1year) = 9.461 \times 10^{15} meters \times \frac{1pc}{3.086 \times 10^{16} meters} = 3.066 \times 10^{-1} pc$$

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ meters} \times \frac{1ly}{9.461 \times 10^{15}} \text{ meters} = 1.581 \times 10^{-5} \text{ ly}$$

1.7 A Brief Review of Trigonometry

PHYS 133 Labs use a small amount of trigonometry, especially triangles that have right angles. Take a look at the right triangle below:
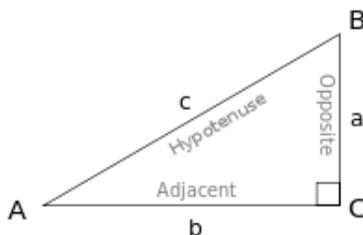
Say we measure angle A. As labeled in the diagram, the edge opposite the right angle is called the hypotenuse, that opposite angle A is called "opposite," and that adjacent to angle A is called "adjacent." Label angle A θ; we can assign Sine, Cosine, and Tangent functions values as follows:

$$\sin \theta = \frac{opposite}{hypotenuse}$$
$$\cos \theta = \frac{adjacent}{hypotenuse}$$
$$\tan \theta = \frac{opposite}{adjacent}$$

where $c^2 = a^2 + b^2$ by the Pythagorian theorem.

In PHYS 133, angles are always measured in *degrees. Make sure your calculator is in degree mode!* To convert from radians to degrees by hand, multiply the number of radians by $\frac{180}{\pi}$; to convert from degrees to radians, multiply the number of degrees by $\frac{\pi}{180}$.

1.8 Statistics

Statistics are frequently one of the most important parts of a measurement, because they reveal information about the reliability of the data. Below, we introduce three different concepts central to statistics and error calculations:

Statistical Error is inherent to the measurement itself. You can think of it as a "random" error. Take for example a package of Skittles: how many pieces are in an average bag? Is there a right answer? We could count the number of Skittles in several bags, but we wouldn't always get the same number; there is some randomness. What if we counted the number of Skittles in a thousand bags and plotted how many times we counted each number? The resulting graph, called a histogram, would look something like this (assuming the existence of partial Skittles):
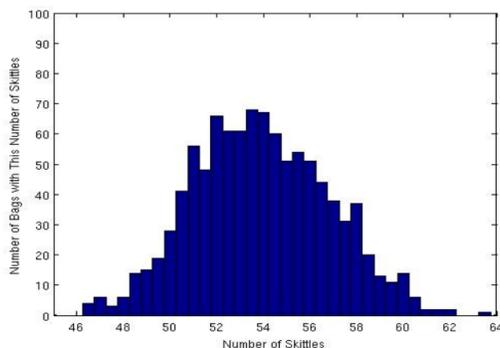
The shape of this graph is called a *bell curve*. A histogram of measurements with statistical error should make a bell curve. According to the graph, the number of Skittles in an average bag is 54 ± 3. The range given by the ± 3, is represented by *error bars* on a graph. If you look at the histogram, you can see the peak of the bell curve is around 54; this is the average number of Skittles per bag. If you look at the width of the histogram (from center to side, halfway to the top), you can see that its about 3. In other words, 54 ± 3 means *On average, there are 54 Skittles in a bag, and most other bags lie within 3 of this number.* $54 + 3 = 57$ is the *upper limit.* $54 - 3 = 51$ is the *lower limit.* Statistical error refers to measurements that are not always the same, but that will appear as a bell curve centered around the mean. More statistical error will result in larger error bars.

Example: Small statistical error: 10.00 ± 0.01, Large statistical error: 10.00 ± 8.00

Systematic Error is an inherent flaw in the experiment or measurement itself. Returning to the Skittles experiment, what if every time we counted the number of Skittles in a bag, we added in an extra five skittles by accident? Here is what the new histogram would look like:
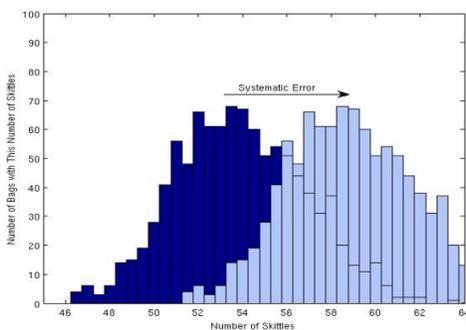
The systematic error shifts the entire sample by five. Our answer is now recorded as 59 ± 3 Skittles per bag.

Calculating Errors: For the purposes of this class, we define the *error* to be the square root of the average of the squared distance between the average and the values themselves. We want to find the average squared difference between the average and our values, and than take the square root of that number. An example is given below:

Lets say we make four measurements: 3, 2, 5, and 2.
The average is $\frac{(3+2+5+2)}{4} = 3$.
The distance between the average (3) and each of the values is: 0, -1, 2, and 0.
The square of these distances is: 0, 1, 4, and 0.
The average of these numbers is: $\frac{(0+1+4+0)}{4} = \frac{5}{4}$.
The square root of this number is 1.118. This is the error in the measurement, which can be written as 3 ± 1.

1.9 A Brief Word on Academic Honesty

The University of Delaware's Code of Conduct clearly outlines a general expectation for students to act honestly and professionally within the classroom. Plagiarism, fabrication, cheating, or any other forms of academic dishonesty will NOT be tolerated, and students found in violation of the University's code will be penalized and reported through the appropriate channels. A brief description of our expectations pertaining to labs are defined below:

Plagiarism: The Code of Conduct defines plagiarism as "the inclusion of some else's words, ideas, images, or data as one's own[3]." Any graded work that results from shared ideas must give credit to all authors involved. Practically, this means that submitted lab reports should contain the names of all lab partners involved. It also requires that any definitions or ideas borrowed from another source be properly quoted and cited. If you are unsure of how to do this, ask your TA's, and we will be happy to review it with you.

Fabrication: This is defined by the Code of Conduct to be "the use of information or the falsification of research or other findings[4]." In lab, this clause prohibits the invention of data, or the falsification of results. Forgery of any kind will also not be tolerated.

Cheating: Cheating is defined in detail in the Code of Conduct, so the details will not be discussed here. We remind you, though, that in lab copying from someone else is equivalent to allowing yourself to be copied from, and will be treated similarly. If you work with someone else, it is perfectly ok to submit a joint paper that is the product of both individuals' thoughts and reflections, as long as both authors recieve credit. It is also acceptible to discuss ideas as a group. However, work that is required to be submitted individually must be written in your own words, even if you discussed the concepts with a partner. Again, if you are unclear on this issue, please ask your TA's for help, and we will be happy to assist you.

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[3]See the University's Code of Conduct at `http://www.udel.edu/stuguide/13-14/code.html#honesty`.

[4]Ibid.

Picture Credits: triangle, `http://upload.wikimedia.org/wikipedia/commons/thumb/4/4f/TrigonometryTriangle.svg/245px-TrigonometryTriangle.svg.png`, histograms from PHYS133 "Measurement and Statistics" lab.

2 Worksheet

One point per problem, 45 problems, 45 points total. *Note this is equivalent to 1.5 regular lab grades.* You only need to turn in the worksheet, which is due **January 12th, 2015**.

2.1 Rounding

Example: Round 324.435 to the nearest ones place. Answer: 324.

1. Round 1895.376 to the nearest tenth

2. Round 709.2353 to the nearest tens place

3. Round 2389.0453 to the nearest hundredth

4. Round 268.982354781 to five significant figures

5. Round 90723.679234 to two significant figures

2.2 Exponent Rules

Compute the following numbers:
Example: $(3 \times 10^4) \times (2 \times 10^2) = 6 \times 10^6$

6. $(3s^3k)^5$

7. $(5^2) \div (10^{-2}) + (3^3)$

8. $(10^{14}) \times (10^{-5}) \div (10^{-10})$

9. $(2^4) \times (2^2) \div (2^8)$

10. $\left(\frac{5h^2 r^6}{9hr^4}\right)^3$

11. $(3c^3 \times 4c^2)^2$

12. $\left(\frac{9^3}{9^5}\right)^3$

2.3   Prefixes

Translate the written-out units attached to the numbers into the same units, but with the proper prefixes attached. You may work with the numbers given; no need to put them into scientific notation.

Example: 240 femtometers = 240 fm.

13. 13 billion years

14. 3.26 lightyears

15. 6536 nanometers

16. 6.023 megameters

17. 75 microseconds

18. 324 centimeters

19. 91 million miles

2.4 Scientific Notation and Significant Figures

Write the following numbers or answers in proper scientific notation, paying attention to the number of significant figures:
Example: $7800 = 7.8 \times 10^3$

20. 3470930000000

21. 0.00007829

22. 1230. (there's a decimal point!)

23. $17.9 \times 2.59 \times 200$

24. $36.18 + 1.6 + 25.1$

25. $97.48 + 63.5 + 29.6426$

26. $(1.592 \times 10^3) \times (18 \times 10^4)$

27. $500 \div 46.420$

2.5   Conversion Factors

28. Convert 5896.4 angstroms to nanometers.

29. Convert 12 AU to centimeters.

30. Convert 30 degrees to radians.

31. Convert $\sqrt{2}/2$ radians to degrees.

32. Convert 16 Megameters to kiloparsecs.

33. Convert 20 years to seconds.

34. Convert 21 parsecs to lightyears.

2.6 Trigonometry

Solve the following word problems, and report your answer in degrees. *Hint: Drawing a diagram might help!*

35. If the opposite side of a right triangle is 3 units and the adjacent side of the same triangle is 4 units, find $\sin(\theta)$, where θ is the angle measured.

36. In a right triangle, one angle is given to be 45 degrees. Calculate $\tan(\theta)$, where θ is the angle measured.

37. In a right triangle, the angle between one side and the hypotenuse is given to be 30 degrees. Calculate $\cos(\theta)$, where θ is the angle measured.

2.7 Statistics

38. If you collected the following data points as part of a measurement, how would you write your final answer? Show your work, following exactly the example given in section 1.8 of this worksheet. The data points are (3, 4, 2, 3, 4, 6, 9, 1).

39. You are an astronomer trying to find a planet. You look for light emitted from the planet's surface, but it is extremely faint and your measurement yields only a few "pieces" of light (called *photons*). You record a measurement of 3 ± 4 photons per second. Have you detected a planet? Why or why not?

40. You decide to use a bigger telescope and detect 4.2 ± 0.3 photons per second. Now have you detected a planet? Why or why not?

2.8 Academic Honesty

Read the attached article entitled "Peering Into Black Holes' Pasts," by Monica Young, and answer the following questions. Be careful to cite your sources!

41. Why do astronomers refer to a black hole "zoo?"

42. What are *active galactic nuclei*?

43. What is the "donut?" Why is it significant?

44. What did Donoso's team discover?

45. What does a black hole eat?

Article Source: Young, Monica. "Peering Into Black Holes' Pasts." Sky and Telescope. Sky Publishing, 24 May 2014. Web. 18 July 2014. http://www.skyandtelescope.com/astronomy-news/peering-black-hole-pasts/. Worksheet created with the help of "Math Worksheets." Math Worksheets. Math-Aids.com, 2009. Web. 29 Dec. 2014. http://www.math-aids.com/.